The V-notch Weir

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Many of Splashback's clients work with water discharge as part of their environmental monitoring. These measurements of flow and discharge are often taken from V-notch weirs. Splashback's team works closely with its clients to provide them the analytical tools to help get the most out of their data. Water flow adds the required context to water quality measurements in many circumstances. Splashback has recently added an Excel function to assist in calculating the discharge from a V-notch weir. We have here provided a summary of the workings of the V-notch weir and the underlying mathematics of the equation used in the Excel function to accompany this release. It is our hope that this article will assist in the accurate use of V-notch weirs as well as give a 'on-site' context to those using the Excel function in the workplace.

INTRODUCTION

Weirs, both permanent and portable, are a common way to enable the standardised measurement of discharge for a water way [1, 2]. Their application is widespread in the field of hydrology as they allow the user to calculate the flow of water from a direct measurement of the water height. As simple as this standard measurement system sounds there are many issues that can prevent a weir measurement from being entirely accurate. There are many weir designs, here we will chiefly discuss one of the most common, the V-notch weir. The overflow section of a V-notch weir is an isosceles triangle with its vertex pointing downwards. V-notch weirs are often chosen over other weir geometries because of their greater accuracy at lower flows. In this article we aim to provide an overview of the weir, specifically the thin plate V-notch weir, and how to correctly utilise it for a discharge measurement. We then provide a derivation of the V-notch discharge equation.

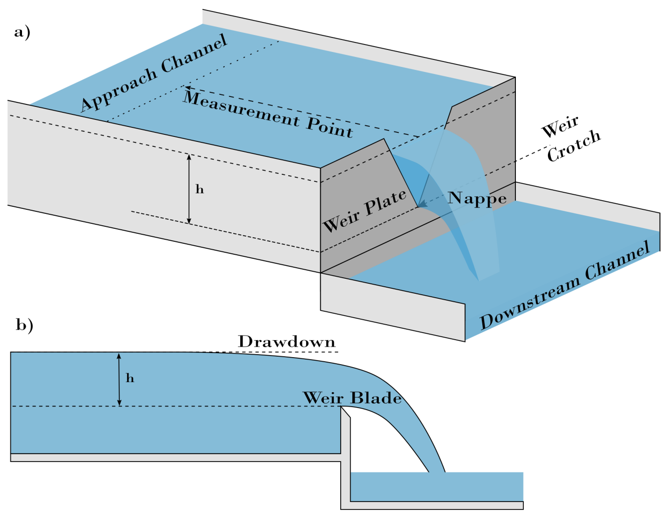

FIG. 1. a) A simple diagram of V-notch weir b) a cross section of the V-notch weir along its long axis. Several important parts of the weir have been labelled.

THE STANDARD WEIR

In constructing a weir there are three important sections to consider; the approach channel upstream of the weir, the weir plate itself and the downstream channel. These all impact the weirs ability to function as a 'standard weir'. Each of these sections have their own impact on the water flow and thus discharge of the weir [2]. Without proper consideration and construction, the weir will not provide accurate discharge data from the standard weir discharge equation, regardless of how accurate the water height measurements may be.

There are several industry standards available [3] that provide detailed requirements for the construction of a standard weir. We will not provide such detail here, but instead offer a discussion of the issues and considerations relating to weir measurement accuracy to assist those striving to obtain accurate discharge data from a weir system. Although we chiefly discuss the V-notch weir design here, many aspects of this discussion are applicable to other weir geometries.

Approach Channel

The approach channel, the section upstream of the weir plate, is used to address the flow characteristics of the water on its approach to the weir plate. Tranquil and calm water is required in the area immediately upstream of the weir plate in order for the discharge to follow the standard discharge equation. The approach channel, being ideally a long, deep, and wide area, allows the water to assume a homogeneous velocity profile. Deviation from an ideal velocity profile is usually visible in the water body as it approaches the weir plate. Upwelling, eddies and other visible abnormalities should be reason to suspect that the weir may not follow the standard discharge equation [4].

Sand or gravel bars building up in the approach channel, or vegetation along the bank, are two possible causes of deviation from ideal conditions [5]. Additionally, eddies can form if the approach channel is not far downstream from a drop or bend, of if the flow section undergoes a rapid expansion. Eddies can concentrate the flow to a narrower section, that is, cause a deviation from ideal conditions.

Rough or turbulent conditions affect both your ability to read the effective height of the water in the weir and also disrupt the discharge relationship [4]. In general the roughness of the surface can be improved by resolving the aforementioned causes. If the water surface is rough due to wind conditions, its likely that the weir will still follow a standard discharge, and only your ability to take an accurate height measurement will be affected. In this case, a stilling well might be used. A wave suppressor, such as an underpass, is one method for improving the water flow characteristic. An underpass forces water through a submerged area below water level directly prior to entering the approach channel, improving the water flow characteristics and reducing the surface roughness. Both stilling wells and wave suppressors can generally be avoided if the approach channel is constructed with the idea of tranquil and uniform flow in mind.

Weir Plate

The weir plate closes the far end of the approach channel, forming the face where the notch, or cut-away, is located. Around this cut-away is the weir blade, over which the water falls. It is named as such due to its sharp profile. The sharp blade is required so that the water flowing through the notch can achieve a smooth, ideally laminar, flow. The weir plate as well as the blade itself can have significant effect on the discharge from the weir.

If the plate is not perpendicular to the walls of the approach channel, and vertical, the resulting calculated discharge may contain non-random offsets in value. If the weir plate is not properly fixed, or its material has some flexibility, the resulting bend with respect to water height may also lead to non-random offsets in the calculated discharge. Both of these phenomenon can be attributed to the flow scenario deviating from that assumed by the standard discharge equation. [1].

The weir blade, if not well kept with regular inspection, may corrode, grow algae and form precipitate or dull and become rounded. Whilst these might appear to be small issues, they too can lead to discharge error, especially at low water heights. Cleaning the weir blade, as part of maintenance, can cause changes in the discharge of the weir. Some time should be given to allow the weir to return to a steady state.

Downstream Channel

The flow of the downstream channel, if restricted, can cause large errors in discharge. Weirs should be designed and placed, such that there is a free flow of water through the notch. That is, that the nappe falls freely down from the weir opening into the downstream channel. If this is not the case the weir is deemed ‘flooded’, and extremely large error can be expected [2]. In general this can be fixed by clearing some blockage downstream of the weir to avoid this backwater build-up. However, in some situations the weir opening may need to be raised.

Other large errors in discharge can occur if the nappe does not freely pass out and over the weir blade, clinging rather to the outer side of the weir plate. This clinging can be due to the weir blade becoming blunt, debris caught in the notch or the water height falling to such a level that the water velocity through the notch cannot sustain a nappe [4].

HEAD MEASUREMENT

While the factors discussed above may impact the discharge of the weir significantly enough to cause noticeable error, it is ultimately an accurate head measurement which governs the success or failure of the instrument. The specific procedure and location of each point of measurement for each individual weir should be known to everyone who undertakes data collection there. As we only give a general overview of measurement procedure, we recommend that staff be informed of the site specific procedure.

A measurement of head in a weir is taken as the water height above the weir crotch. However, this height is not the water height directly above the crotch itself but rather the height at a location upstream from the weir plate. The reason for this is the section of water in and around the weir crotch has begun to fall and accelerate through the notch [1, 2]. This section of water is called the drawdown and a measurement of water height in this area does not reflect the true water height in the weir. The height change in the drawdown is directly related to the velocity gained by the water exiting the notch.

The location of the head measurement should be at a distance of 4 to 6 times the head upstream from the weir crotch to avoid the drawdown. However, going too far upstream may mean that the gradient of the stream or upstream section causes a larger head measurement. The exact location in each site should take into account these factors. The use of staff gauges along the bankside should be investigated on a case by case basis, the misplacement of which can lead to systematic offsets in data.

When taking head data from a weir it is important to take multiple measurements over a period of time. An instantaneous measurement may reflect the discharge of a surge in water rather than the average flow. An average of several measurements should be taken over a period of time, reducing both error in the head measurement and error due to surges [4].

THE V-NOTCH

Owing to its geometry the V-notch weir is often chosen over the rectangular weir due to its performance at both low and high flow. Low flow performance of the V-notch is greater as the smaller area closer to the vertex or crotch of the notch allows for a nappe to more readily form. Low flow of water through a rectangular notch weirs often causes the water to stick to the weir plate as the velocity of the water does not allow for the formation of a nappe.

V-notches come in a variety of internal angles, this being the angle between the weir blades at the weir crotch. The angle chosen should reflect the desired measurement range or expected flow through the weir. Weirs that measure low flow should choose a notch with a tighter angle, those with higher flow a larger angle. Common notch angles are $\theta = 90°$, $\theta = 53.13°$ and $\theta = 28.07°$ [3]. These corresponding to numbers that simplify the discharge equation allowing the $\tan(\theta/2)$ term to take the values 1, 0.5 and 0.25 respectively.

DISCHARGE EQUATION

The V-notch weir has been the subject of extensive study, constraining the equation relating its effective height to discharge to a relatively simple form [1]. The equation relating the 'Head' to the discharge of water for a V-notch weir is expressed as the following

$$Q = \frac{8}{15} C_e \sqrt{2g} \tan \frac{\theta}{2} H^{\frac{5}{2}} \tag{1}$$

Where Q is the discharge, g is acceleration due to gravity, C_e is the discharge coefficient, θ is the angle of the V-notch and H is the head.

In Splashback's Excel function, `SBK.V_NOTCH(water level,angle)`, we use this equation with the inputs of water height (in meters) and V-notch angle angle (in degrees) to generate the flow in Cumecs through the weir. This function takes any value of angle from 20° to 100°. The coefficient of discharge in this calculation is derived internally from the measurement of height and the angle with the assumption that the weir is performing in a standard way. It is important that the weir is performing as such when making the head measurements this function is applied to.

Via inspection of this equation, you can see that the accuracy of your calculated flow data is reliant on both accurate effective head measurements and an accurate value for the coefficient of discharge. The discharge coefficient is an experimentally determined coefficient dependent on a range of parameters relating to the weir and approach channel. Despite the complexity of its dependencies, the discharge coefficient has been widely investigated and now can be found in many standards on weir measurement in the form of either graphs or tables.

In forming a mathematical model of something as changeable as water flow, assumptions as to the 'state' of the system must be made. The general assumption that this model makes is that the flow of water in the approach channel is tranquil and uniform. When this is not the case the discharge of the weir no longer follows the given equation. The discussion of different conditions in the texts referenced demonstrates the sensitivity of such a system to any deviation from ideal. Bellow follows a derivation of the standard V-notch weir discharge equation.

Discharge Equation Derivation

The discharge, Q, of a fluid through an area is given by the velocity, ν, of the fluid multiplied by the area, A, through which it passes.

$$Q = \nu \times A. \tag{2}$$

However, in this case both fluid velocity and area are dependent on the depth of the fluid. In this calculation we cannot simply multiply the area of the V-notch by some scalar to arrive at discharge, we must integrate the velocity profile over the area with respect to height. The relationship between the waters velocity and its depth is given by

$$\nu = C_e\sqrt{2gh}, \tag{3}$$

Where g is acceleration due to gravity and h is water depth. To account for the friction effects between the water and the area it is passing through we have multiplied by a constant C_e, this being the coefficient of discharge.

Now consider the area of the V-notch. Each 'slice' from top to bottom will have uniform velocity across it. Figure 2 depicts our method. Each slice will have the small area, dA, of

$$\begin{aligned} \mathrm{d}A &= 2l\ \mathrm{d}h \qquad (4)\\ &= 2(H-h)\tan\left(\frac{\theta}{2}\right)\mathrm{d}h \qquad (5)\end{aligned}$$

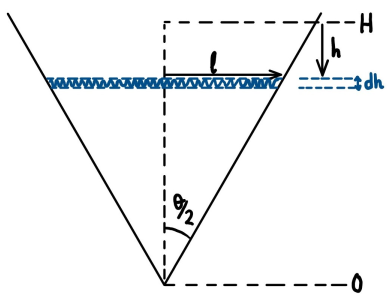

FIG. 2. The V-notch area broken up into slices of dh thickness.

Combining this area with our expression for velocity yields the discharge of water through that slice

$$\begin{aligned} \mathrm{d}Q &= \nu \times \mathrm{d}A \qquad (6)\\ &= 2C_e\sqrt{2g}\tan\left(\frac{\theta}{2}\right)(H-h)\sqrt{h}\ \mathrm{d}h \qquad (7)\end{aligned}$$

Integrating this expression over the water height $0 \to H$ yeilds the total discharge.

$$\begin{aligned} Q &= \int_0^H 2C_e\sqrt{2g}\tan\left(\frac{\theta}{2}\right)(H-h)\sqrt{h}\ \mathrm{d}h \qquad (8)\\ &= 2C_e\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\int_0^H (H-h)\sqrt{h}\ \mathrm{d}h \qquad (9)\\ &= 2C_e\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\left[\frac{2}{3}Hh^{\frac{3}{2}} - \frac{2}{5}H^{\frac{5}{2}}\right]_0^H \qquad (10)\\ &= 2C_e\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\left[\frac{2}{3}H^{\frac{5}{2}} - \frac{2}{5}H^{\frac{5}{2}}\right] \qquad (11)\\ &= \boxed{\frac{8}{15}C_e\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{\frac{5}{2}}} \qquad (12)\end{aligned}$$

Arriving at the standard V-notch weir discharge equation.

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[2] A. J. Clemmens, T. L. Wahl, M. G. Bos, J. A. Replogle *Water Measurement with Flumes and Weirs*, pp. 59–164. International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands, ISBN 90-70754-55-X (2001).

[3] ASTM International, *Standard Test Method for Open-Channel Flow Measurement of Water with Thin-Plate Weirs*, ASTM Designation: D 5242 – 92 (Reapproved 2001).

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